Topic 06 - Completing the Square Solutions

Section A

Write the following expressions in complete square form (i.e. in the form $(x + a)^2 + b$)

1. $x^2 + 2x$
2. $x^2 - 8x$
3. $x^2 + 4x + 3$
4. $x^2 - 6x + 4$
5. $x^2 - 10x - 1$
6. $x^2 + 3x$
7. $x^2 - 7x$
8. $x^2 + 7x - 4$
9. $x^2 - 5x - 5$
10. $x^2 + 7x + \frac{1}{2}$

1/ $(x+1)^2 - 1$

2/ $(x-4)^2 - 16$

3/ $(x+2)^2 - 4 + 3$

$(x+2)^2 - 1$

4/ $(x-3)^2 - 9 + 4$

$(x-3)^2 - 5$

5/ $(x-5)^2 - 25 - 1$

$= (x-5)^2 - 26$

6/ $(x+\frac{3}{2})^2 - \frac{9}{4}$

7/ $(x-\frac{7}{2})^2 - \frac{49}{4}$

8/ $(x+\frac{7}{2})^2 - \frac{49}{4} - 4$

$= (x+\frac{7}{2})^2 - \frac{49}{4} - \frac{16}{4}$

$= (x+\frac{7}{2})^2 - \frac{65}{4}$

9/ $(x-\frac{5}{2})^2 - \frac{25}{4} - 5$

$= (x-\frac{5}{2})^2 - \frac{25}{4} - \frac{20}{4}$

$= (x-\frac{5}{2})^2 - \frac{45}{4}$

10/ $(x+\frac{7}{2})^2 - \frac{49}{4} + \frac{1}{2}$

$= (x+\frac{7}{2})^2 - \frac{49}{4} + \frac{2}{4}$

$= (x+\frac{7}{2})^2 - \frac{47}{4}$

Section B

Q1 (Jun 2005, Q2)

(i) Express $3x^2 + 12x + 7$ in the form $3(x + a)^2 + b$. [4]

(ii) Hence write down the equation of the line of symmetry of the curve $y = 3x^2 + 12x + 7$. [1]

i/ $3[x^2 + 4x] + 7 = 3[(x+2)^2 - 4] + 7 = 3(x+2)^2 - 12 + 7$

$= 3(x+2)^2 - 5$

ii/ $x = -2$

Q2 (Jun 2006, Q3)

(i) Express $2x^2 + 12x + 13$ in the form $a(x + b)^2 + c$. [4]

(ii) Solve $2x^2 + 12x + 13 = 0$, giving your answers in simplified surd form. [3]

i/ $2[x^2 + 6x] + 13 = 2[(x+3)^2 - 9] + 13 = 2(x+3)^2 - 18 + 13$

$= 2(x+3)^2 - 5$

ii/ $2(x+3)^2 - 5 = 0 \Rightarrow 2(x+3)^2 = 5 \Rightarrow (x+3)^2 = \frac{5}{2}$

$\Rightarrow x + 3 = \pm\sqrt{\frac{5}{2}}$ $\left(\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}\right)$

$\Rightarrow x + 3 = \pm\frac{\sqrt{10}}{2} \Rightarrow x = -3 \pm \frac{\sqrt{10}}{2}$

Q3 (Jan 2007, Q6)

(i) Express $2x^2 - 24x + 80$ in the form $a(x - b)^2 + c$. [4]

(ii) State the equation of the line of symmetry of the curve $y = 2x^2 - 24x + 80$. [1]

(iii) State the equation of the tangent to the curve $y = 2x^2 - 24x + 80$ at its minimum point. [1]

i/ $2[x^2 - 12x] + 80 = 2[(x-6)^2 - 36] + 80$

$= 2(x-6)^2 - 72 + 80$

$= 2(x-6)^2 + 8$

ii/ $x = 6$

iii/ $y = 8$

Q4 (Jun 2008, Q10) [Modified]

(i) Express $2x^2 - 6x + 11$ in the form $p(x + q)^2 + r$. [4]

(ii) State the coordinates of the vertex of the curve $y = 2x^2 - 6x + 11$. [2]

i/ $2[x^2 - 3x] + 11 = 2[(x - \frac{3}{2})^2 - \frac{9}{4}] + 11$

$= 2(x - \frac{3}{2})^2 - \frac{9}{2} + \frac{22}{2}$

$= 2(x - \frac{3}{2})^2 + \frac{13}{2}$

ii/ $(\frac{3}{2}, \frac{13}{2})$

Q5 (Jan 2009, Q6) [Modified]

(i) Express $5x^2 + 20x - 8$ in the form $p(x+q)^2 + r$. **[4]**

(ii) State the equation of the line of symmetry of the curve $y = 5x^2 + 20x - 8$. **[1]**

i/ $5[x^2 + 4x] - 8 = 5[(x+2)^2 - 4] - 8$

$= 5(x+2)^2 - 20 - 8$

$= 5(x+2)^2 - 28$

ii/ $x = -2$

Q6 (Jun 2012, Q4)

(i) Express $2x^2 - 20x + 49$ in the form $p(x-q)^2 + r$. **[4]**

(ii) State the coordinates of the vertex of the curve $y = 2x^2 - 20x + 49$. **[2]**

i/ $2[x^2 - 10x] + 49 = 2[(x-5)^2 - 25] + 49$

$= 2(x-5)^2 - 50 + 49$

$= 2(x-5)^2 - 1$

ii/ $(5, -1)$